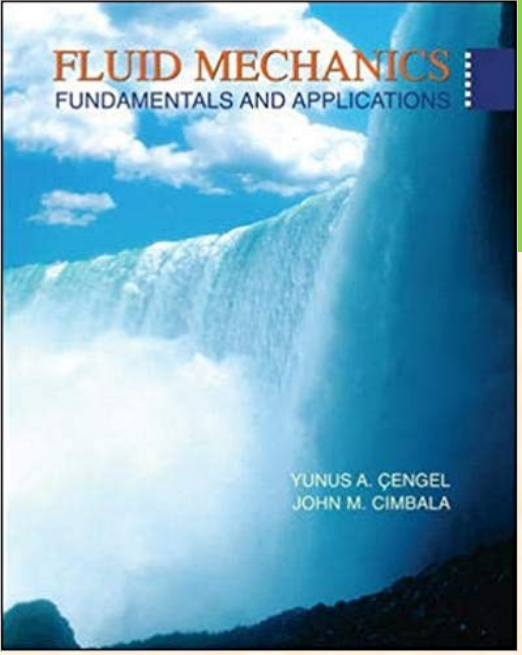

YUNUS A. ÇENGEL and JOHN M. CIMBALA, "Fluid Mechanics: Fundamentals and Applications",1st ed., McGraw-Hill,2006.

Course name

Incompressible Fluid Mechanics

Lecture-02- chapter-03

Fluid flow concept and Basic equations

Lecture slides by

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Outline

- ***Static, Dynamic, and Stagnation Pressures***
- ***Stagnation Pressures***
- ***Flow measurements:***
- **(1) Pitot tube**
  - ✓ ***open system***
  - ✓ ***closed system***
- **(2) Orifice meter**
  - ✓ ***open system***
  - ✓ ***closed system***
- **(3) Venturi meters**
- ***Examples***
- ***Homeworks:***

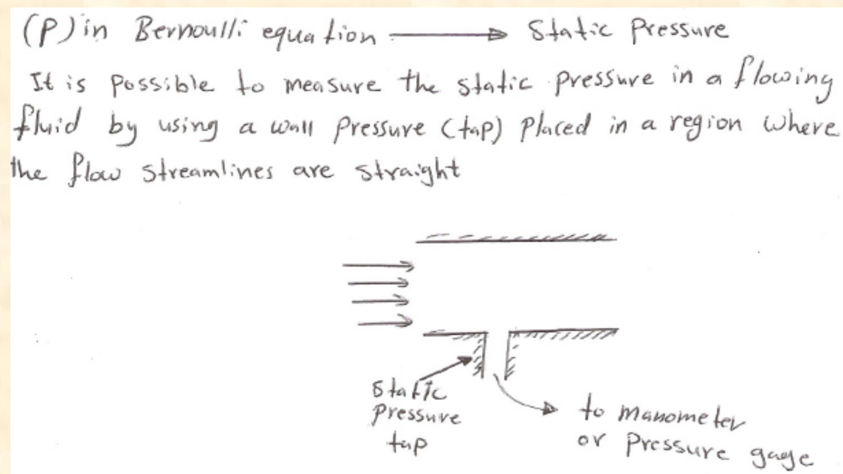
3.7 Static, Dynamic, and Stagnation Pressures

The **Bernoulli equation** states that the sum of the flow, kinetic, and potential energies of a fluid particle along a streamline is constant. ***Therefore, the kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow**, **causing the pressure to change***. This phenomenon can be made more visible by multiplying the Bernoulli equation by the density ρ,

$$P + \rho \frac{V^2}{2} + \rho g z = \text{constant (along a streamline)}$$

Each term in this equation has pressure units, and thus each term represents some kind of pressure:

- P is the **static pressure** (it does not incorporate any dynamic effects); it represents the actual thermodynamic pressure of the fluid. This is the same as the pressure used in thermodynamics and property tables.
- $\rho V^2/2$ is the **dynamic pressure;** it represents the pressure rise when the fluid in motion is brought to a stop isentropically.
- ρgz is the **hydrostatic pressure,** which is not pressure in a real sense since its value depends on the reference level selected; it accounts for the elevation effects, i.e., of fluid weight on pressure.



3.7 Stagnation Pressures

is obtained when a flowing fluid is decelerated to zero speed by frictionless process.

Stagnation pressure is measured in a lab by using probe with a hole that faces directly upstream as shown in the figure

$$P_{stag} = P + \rho \frac{V^2}{2}$$

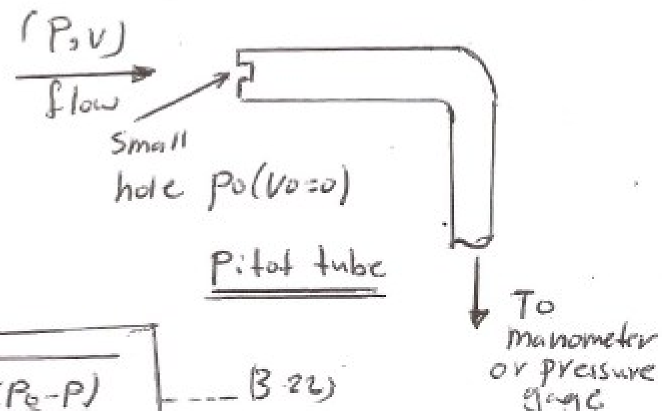
$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$

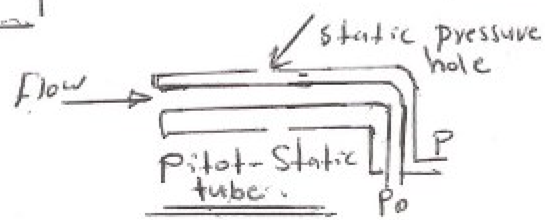
By applying B-E between point upstream the Pitot tube where the pressure (P) and velocity (V) and point (o)

$$\frac{P}{\gamma} + \frac{V^2}{2g} = \frac{P_o}{\gamma} + \frac{V_o^2}{2g}$$

$$P_o = P + \frac{1}{2}\rho V^2 \Longrightarrow \boxed{V = \sqrt{\frac{2(P_o - P)}{\rho}}} \quad \text{--- (3-22)}$$

Dynamic Pressure





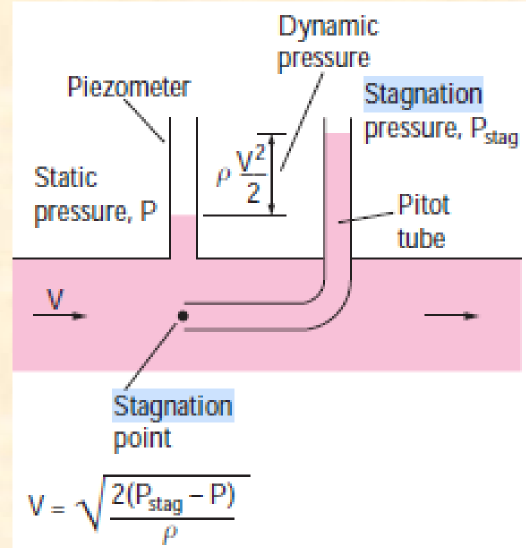


FIGURE 5–27
The static, dynamic, and stagnation pressures.

- The sum of the static and dynamic pressures is called the **stagnation Pressure**
- The **stagnation pressure** represents the pressure at a point where the fluid is brought to a complete stop isentropically. The static, dynamic, and stagnation pressures are shown in Fig. 5–27. When static and stagnation pressures are measured at a specified location, the fluid velocity at that location can be calculated from the velocity Eq.:

3.8 Flow measurements: (1) Pitot tube

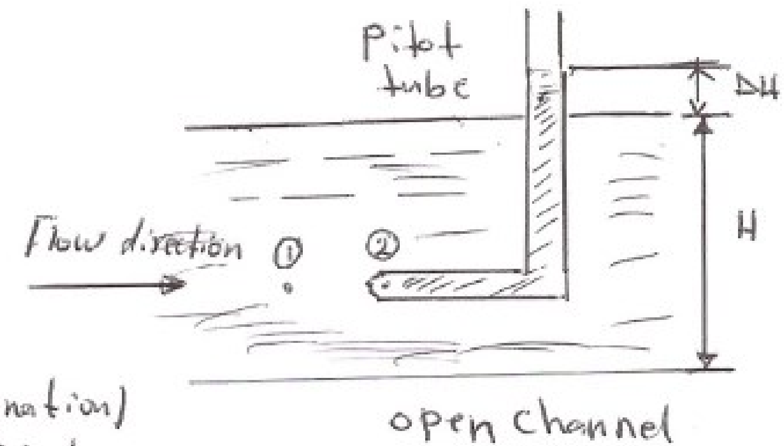
- A **Pitot tube** is a small tube with its open end aligned *into* the flow so as to sense the full impact pressure of the flowing fluid. It measures the stagnation pressure.
- However, the static pressure tap and Pitot tube can still be used, but they must be connected to some other kind of pressure measurement device such as a U-tube manometer or a pressure transducer

1- Pitot tube

(a) used in open channel

B.E between 1 & 2

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2$$

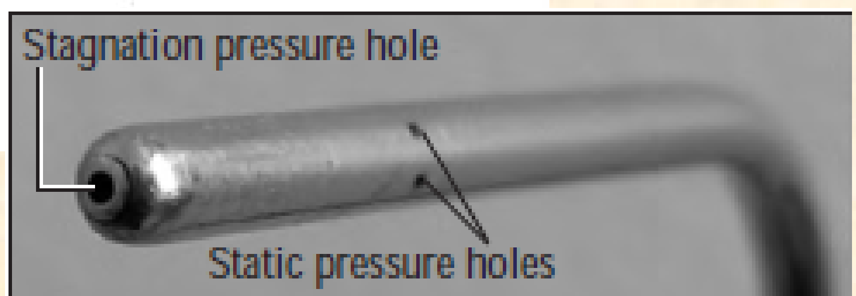


$P_1=\gamma H$, $Z_1=Z_2$, $V_2=0$ (stagnation point)

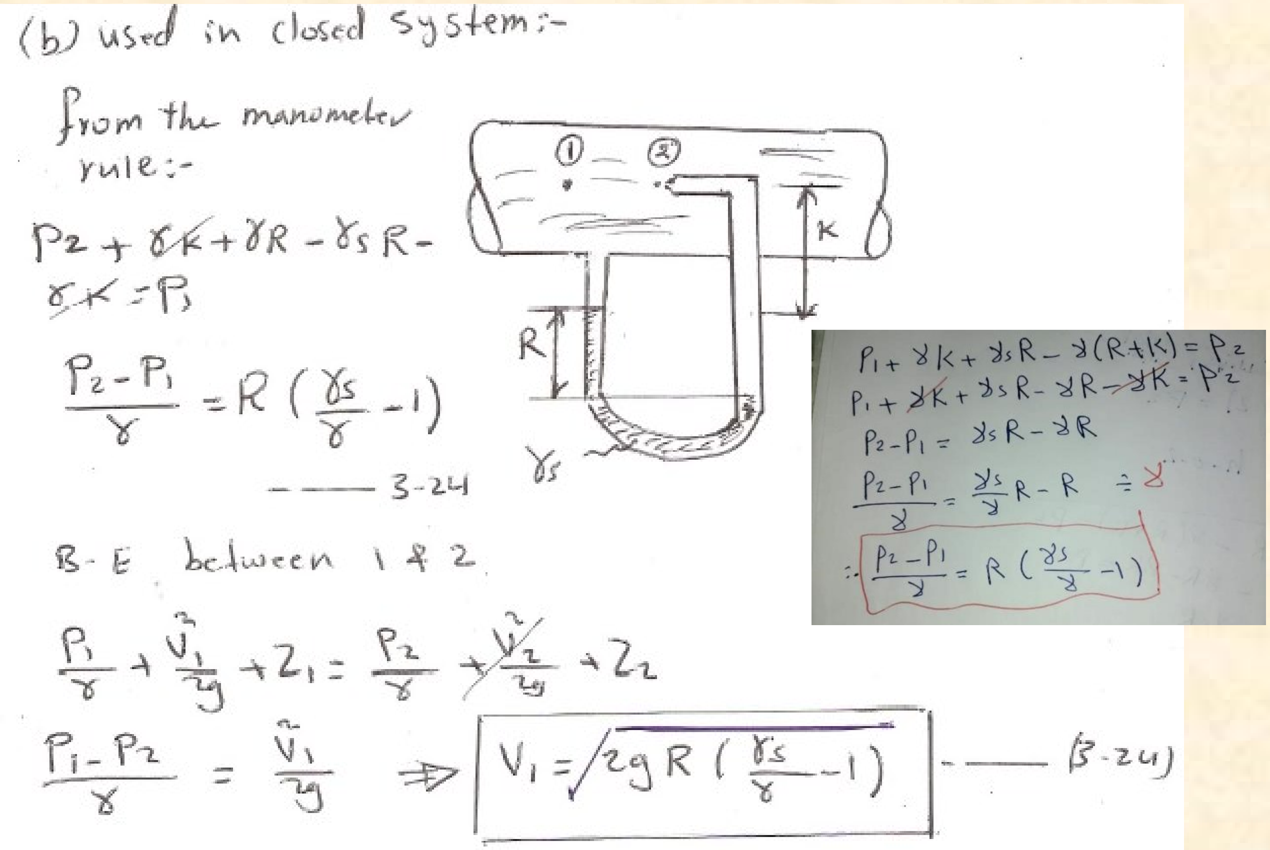
$P_2=\gamma(H+\Delta H)$

$$\therefore \frac{\gamma H}{\gamma}+\frac{V_1^2}{2g}=\frac{\gamma(H+\Delta H)}{\gamma} \longrightarrow \frac{V_1^2}{2g}=H+\Delta H-H$$

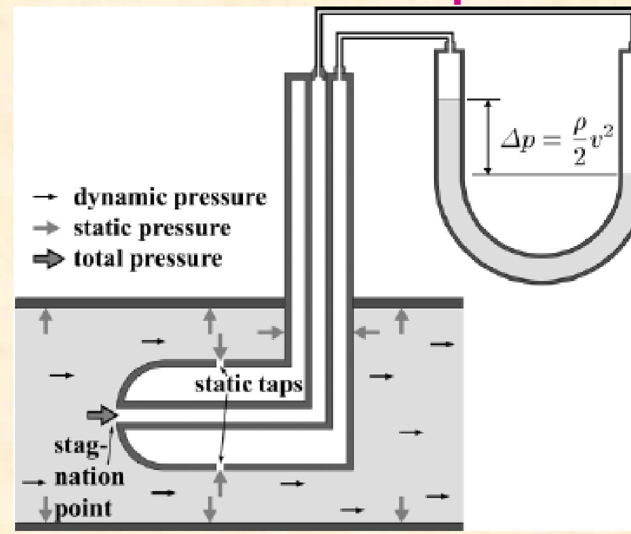
$$\therefore V_1=\sqrt{2g\Delta H} \qquad \text{—— 3.23}$$

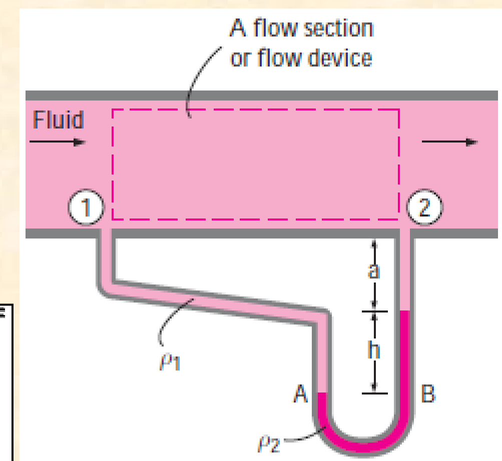


3.8 Flow measurements: Manometer with Pitot tube

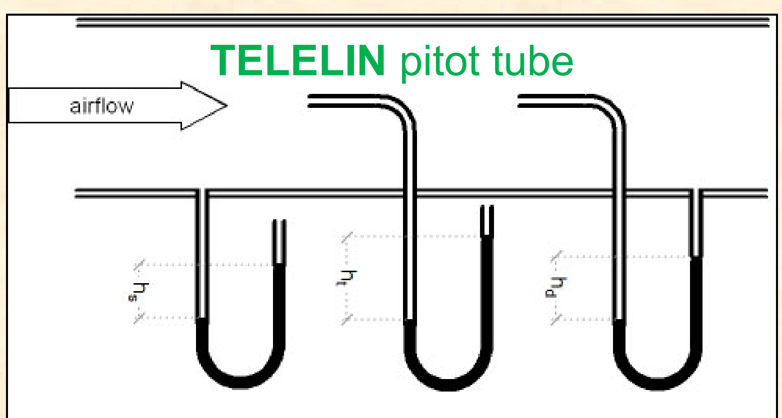


Manometer examples



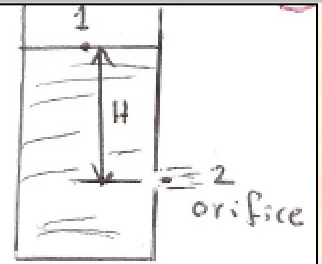


- Sven Eckert and et als., "Velocity Measurement Techniques for Liquid Metal Flows", January 2007,DOI: 10.1007/978-1-4020-4833-3_17, book chapter Dresden, Germany.
- http://pitottubes.com/pitot-tube-manometer.php



3.8 Flow measurements: (1) Orifice meter in open system

(a) used in tanks



B.E 1/2

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2$$

$Z_1-Z_2=H$

$P_1=P_2=P_{atm}$

$V_1 \ll V_2 \quad \therefore V_1\simeq 0$

$$\therefore \frac{V_2^2}{2g}=H \Rightarrow \boxed{V_2=\sqrt{2gH}} \quad \text{——— (3-25)}$$

Example problem: Emptying of a tank

For free jets → $P=P_{atm}$

Apply Bernoulli Equation between free surface (1) and exit (2)

$$P_1+\frac{1}{2}\rho V_1^2+\rho g z_1 = P_2+\frac{1}{2}\rho V_2^2+\rho g z_2 \; (z_2 \to 0)$$

$$P_{atm}+\frac{1}{2}\rho V_1^2+\rho g h = P_{atm}+\frac{1}{2}\rho V_2^2+0$$

cons. of mass

$A_tV_1=A_eV_2$

$V_1=\left(\frac{A_e}{A_t}\right)V_2$

let say $D_e=\frac{1}{10}D_t$

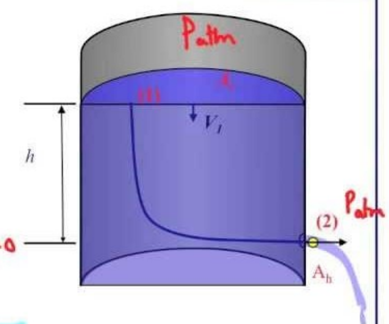
$\frac{A_e}{A_t}=\left(\frac{D_e}{D_t}\right)^2=\frac{1}{100}$

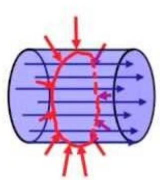
$\left(\frac{A_e}{A_t}\right)^2=\frac{1}{10000}$

$$\frac{1}{2}\rho\left(\frac{A_e}{A_t}\right)^2V_2^2+\rho g h=\frac{1}{2}\rho V_2^2 \rightarrow \boxed{V_2=\sqrt{2gh}}$$

small in comparison... due to $\frac{A_e}{A_t}\ll 1$

$z=0$





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https://www.youtube.com/watch?v=QiWsX9JRb6c
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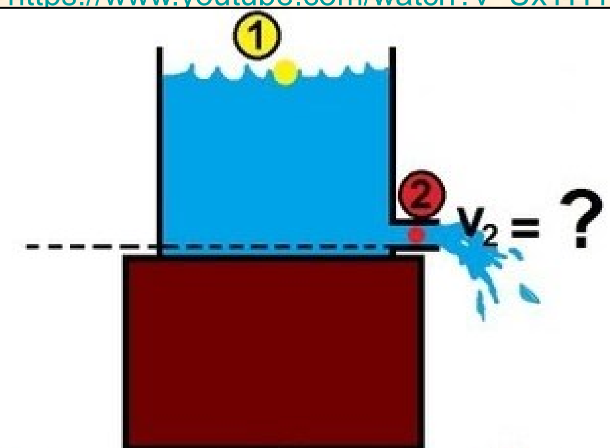


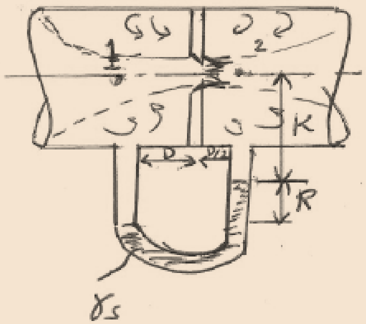
$$\cancel{P_1}+\rho g h_1+\cancel{\frac{1}{2}\rho v_1^2}^{\,0}=\cancel{P_2}+\cancel{\rho g h_2}^{\,0}+\frac{1}{2}\rho v_2^2$$

3.8 Flow measurements: (2) Orifice meter in closed system

(b) used in closed system

B-E 1,2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 \quad \text{--- 3-26a}$$



continuity equation

$$A_1 V_1 = A_2 V_2$$

A_2: vena contraction area

$$A_2 = C_c A_o \quad \text{---- 3-26b}$$

C_c: contraction coefficient (معامل التقلص)

A_o: orifice Area

∴ equ 3-26b becomes

$$A_1 V_1 = C_c A_o V_2$$

$$V_1 = C_c \frac{\frac{\pi}{4} D_o^2}{\frac{\pi}{4} D_1^2} V_2$$

$$\boxed{V_1 = C_c \left(\frac{D_o}{D_1}\right)^2 V_2} \quad \text{--- 3-27}$$

∴ equation 3-26 a

$$\frac{P_1 - P_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g} \quad \text{--- 3-28}$$

∴ Sub. equation 3-27 into 3-28

$$V_2 = \sqrt{\frac{2g\left(\frac{P_1 - P_2}{\gamma}\right)}{1 - C_c^2 \left(\frac{D_o}{D_1}\right)^4}} \quad \text{--- 3-29}$$

Consider incompressible steady flow of a fluid in a horizontal pipe of diameter D that is constricted to a flow area of diameter d, as shown in Fig. 8–55. The mass balance and the Bernoulli equations between a location before the constriction (point 1) and the location where constriction occurs (point 2) can be written as

Mass balance: $\dot{V} = A_1V_1 = A_2V_2 \quad \rightarrow \quad V_1 = (A_2/A_1)V_2 = (d/D)^2V_2$ **(8–68)**

Bernoulli equation ($z_1 = z_2$): $\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$ **(8–69)**

Combining Eqs. 8–68 and 8–69 and solving for velocity V_2 gives

Obstruction (with no loss): $V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}}$ **(8–70)**

where $\beta = d/D$ is the diameter ratio. Once V_2 is known, the flow rate can be determined from $\dot{V} = A_2V_2 = (\pi d^2/4)V_2$.

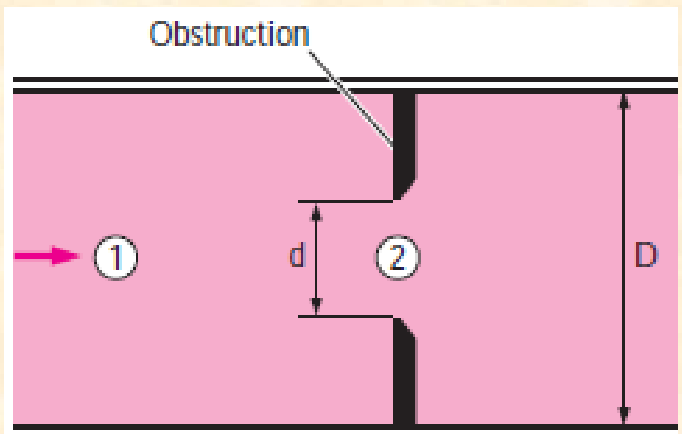


FIGURE 8–55
Flow through a constriction in a pipe.

3.8 Flow measurements: (2) Orifice meter in closed system

From manometer rule:-

$$\frac{P_1 - P_2}{\gamma} = R\left(\frac{\gamma_s}{\gamma} - 1\right)$$

$$\therefore V_2 = \sqrt{\frac{2gR\left(\frac{\gamma_s}{\gamma} - 1\right)}{1 - C_c^2\left(\frac{D_o}{D_1}\right)^4}} \qquad \text{—— 3-30}$$

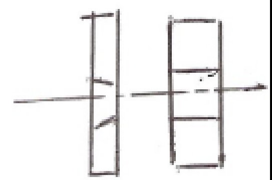
$Q = AV$

$V_{actual} = C_v V_{th}$

$A_2 = C_c A_o$

$\therefore Q_{actu} = C_v V_{th} C_c A_o$

$Q_{act} = C_d V_{th} A_o$



Cd: discharge coefficient

$C_d = C_c \cdot C_v$

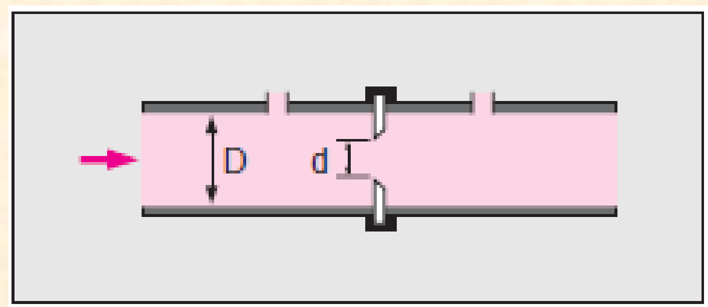
Cv: Coefficient of velocity

$$C_d = \frac{Q_{actual}}{Q_{th}}$$

$$C_c = \frac{A_{orifice}}{A_{vena}}$$

The fluid stream will continue to contract past the obstruction, and the vena contracta area is less than the flow area of the obstruction. Both losses can be accounted for by incorporating a correction factor called the **discharge coefficient *Cd*** whose value (which is less than1) is determined experimentally. Then the flow rate for obstruction flowmeters can be expressed as:-

$$\dot{V} = A_0 C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}}$$



(a) Orifice meter

3.8 Flow measurements: (3) Venturi meters

The Venturi meter, invented by the American engineer Clemans Herschel (1842–1930) and named by him after the Italian Giovanni Venturi (1746– 1822) for his pioneering work on conical flow sections, is the most accurate flowmeter in this group, but it is also the most expensive.

$C_c = 1$

B-E 1, 2

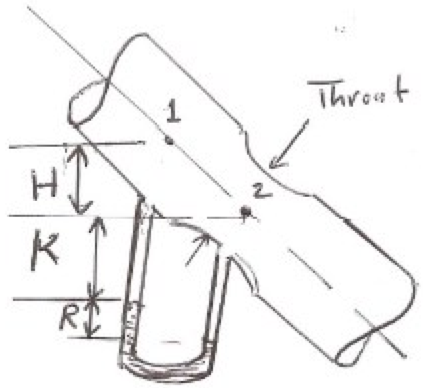
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 \quad \text{---- 3-31}$$

Continuity equ

$$A_1 V_1 = A_2 V_2 \quad \text{--- 3-32}$$

From 3-31 & 3-32

$$V_2 = \sqrt{\frac{2g\left(\frac{P_1 - P_2}{\gamma} + H\right)}{1 - \left(\frac{D_2}{D_1}\right)^4}} \quad \text{--- 3-33}$$



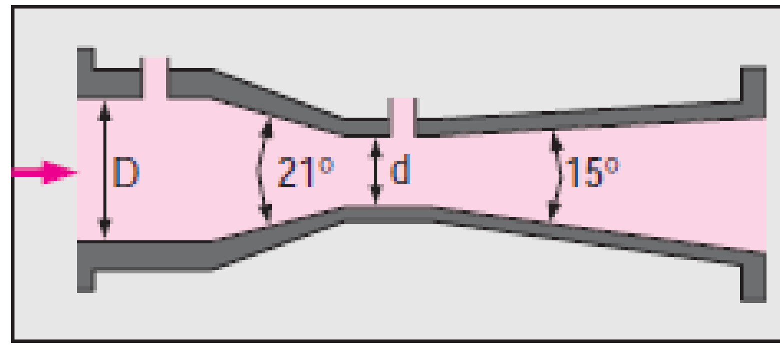
From the manometer

$$P_1 + \gamma H + \gamma K + \gamma_s R = P_2 + \gamma K + \gamma R$$

$$\frac{P_1 - P_2}{\gamma} + H = R\left(\frac{\gamma_s}{\gamma} - 1\right) \quad \text{--- 3-34}$$

Sub 3-34 into 3-33

$$V_2 = \sqrt{\frac{2gR\left(\frac{\gamma_s}{\gamma} - 1\right)}{1 - \left(\frac{D_2}{D_1}\right)^4}} \quad \text{--- 3-35}$$



(c) Venturi meter

***Cd* = 0.98 for Venturi meters**

Examples

***Example $_1$* : Velocity Measurement by a Pitot Tube**

A piezometer and a Pitot tube are tapped into a horizontal water pipe, as shown in Fig. 5–41, to measure static and stagnation (static + dynamic) pressures. For the indicated water column heights, determine the velocity at the center of the pipe.

Sol.$_1$:

$$P_1 = \rho g(h_1 + h_2)$$

$$P_2 = \rho g(h_1 + h_2 + h_3)$$

Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}^{\nearrow 0} + z_2 \quad \rightarrow \quad \frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g}$$

Substituting the P_1 and P_2 expressions gives

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g} = \frac{\rho g(h_1 + h_2 + h_3) - \rho g(h_1 + h_2)}{\rho g} = h_3$$

Solving for V_1 and substituting,

$$V_1 = \sqrt{2gh_3} = \sqrt{2(9.81 \text{ m/s}^2)(0.12 \text{ m})} = \mathbf{1.53 \text{ m/s}}$$

Discussion Note that to determine the flow velocity, all we need is to measure the height of the excess fluid column in the Pitot tube.

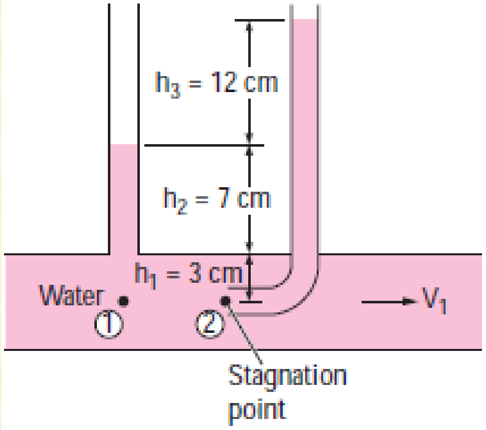


FIGURE 5–41
Schematic for Example 5–8.

Examples

Example$_2$: A Pitot-static probe connected to a water manometer is used to measure the velocity of air. If the deflection (the vertical distance between the fluid levels in the two arms) is 7.3 cm, determine the air velocity. Take the density of air to be 1.25 kg/m^3.

Sol.$_2$: ***Properties:***density of fluids are ρ_w = 1000 kg/m^3 & ρ_a = 1.25 kg/m^3.

Note: that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

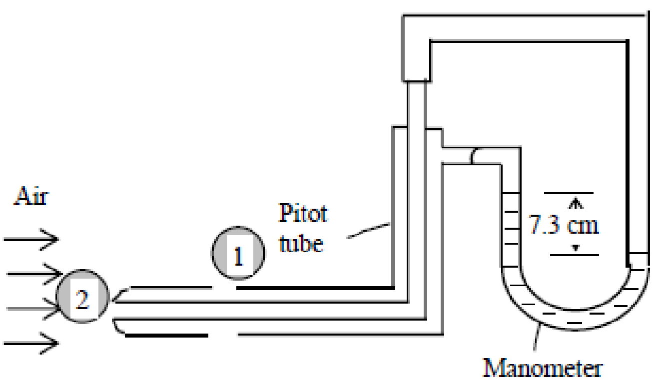
$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \rightarrow \frac{P_1}{\rho g}+\frac{V_1^2}{2g}=\frac{P_2}{\rho g} \rightarrow V_1=\sqrt{\frac{2(P_2-P_1)}{\rho_{air}}} \quad (1)$$

The pressure rise at the tip of the Pitot-static probe is simply the pressure change indicated by the differential water column of the manometer,

$$P_2-P_1=\rho_{\text{water}}gh \quad (2)$$

Combining Eqs. (1) and (2) and substituting, the flow velocity is determined to be

$$V_1=\sqrt{\frac{2\rho_{\text{water}}gh}{\rho_{\text{air}}}}=\sqrt{\frac{2(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.073\text{ m})}{1.25\text{ kg/m}^3}}=\mathbf{33.8\ m/s}$$



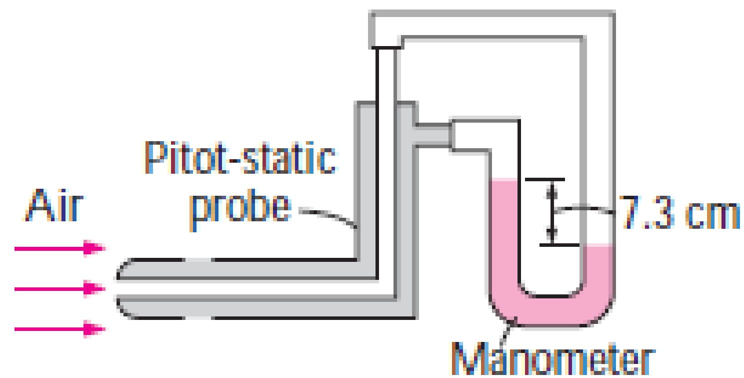


FIGURE P5–60

Discussion Note that flow velocity in a pipe or duct can be measured easily by a Pitot-static probe by inserting the probe into the pipe or duct parallel to flow, and reading the differential height. Also note that this is the velocity at the location of the tube. Several readings at several locations in a cross-section may be required to determine the mean flow velocity.

Examples

Example$_3$: Measuring Flow Rate with an Orifice Meter. The flow rate of methanol at 20°C (ρ=788.4 kg/m^3 and μ= 5.857*10^{-4} kg/m · s) through a 4-cm-diameter pipe is to be measured with a 3-cm-diameter orifice meter equipped with a mercury manometer across the orifice place, as shown in Fig. 8–60. If the differential height of the manometer is read to be 11 cm, determine the flow rate of methanol through the pipe and the average flow velocity.

Sol.$_3$: The flow rate of methanol is to be measured with an orifice meter. For a given pressure drop across the orifice plate, the flow rate and the average flow velocity are to be determined. ***Assumptions*** **1** The flow is steady and incompressible. **2** The discharge coefficient of the orifice meter is *Cd* =0.61.

Properties The density and dynamic viscosity of methanol are given to be ρ = 788.4 kg/m^3 and μ = 5.857 × 10^{-4} kg/m · s, respectively. We take the density of mercury to be 13,600 kg/m^3.

Analysis The diameter ratio and the throat area of the orifice are

$$\beta = \frac{d}{D} = \frac{3}{4} = 0.75$$

$$A_0 = \frac{\pi d^2}{4} = \frac{\pi(0.03\text{ m})^2}{4} = 7.069 \times 10^{-4}\text{ m}^2$$

The pressure drop across the orifice plate can be expressed as

$$\Delta P = P_1 - P_2 = (\rho_{Hg} - \rho_{met})gh$$

Then the flow rate relation for obstruction meters becomes

$$\dot{V} = A_0C_d\sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} = A_0C_d\sqrt{\frac{2(\rho_{Hg} - \rho_{met})gh}{\rho_{met}(1-\beta^4)}} = A_0C_d\sqrt{\frac{2(\rho_{Hg}/\rho_{met} - 1)gh}{1-\beta^4}}$$

Substituting, the flow rate is determined to be

$$\dot{V} = (7.069 \times 10^{-4}\text{ m}^2)(0.61)\sqrt{\frac{2(13{,}600/788.4 - 1)(9.81\text{ m/s}^2)(0.11\text{ m})}{1 - 0.75^4}}$$

$$= \mathbf{3.09 \times 10^{-3}\ m^3/s}$$

which is equivalent to 3.09 L/s. The average flow velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{3.09 \times 10^{-3}\text{ m}^3/\text{s}}{\pi(0.04\text{ m})^2/4} = \mathbf{2.46\ m/s}$$

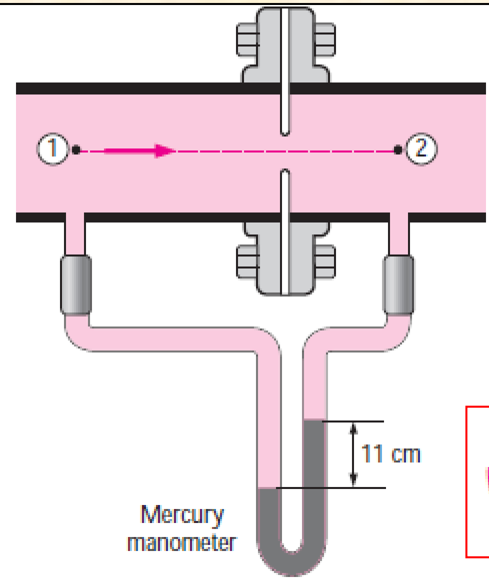


FIGURE 8–60
Schematic for the orifice meter considered in Example 8–10.

$$\dot{V} = A_0C_d\sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$P_1 + \gamma_m h - \gamma_{Hg} h = P_2 \qquad \gamma = \rho g$$

$$P_1 - P_2 = h(\gamma_{Hg} - \gamma_m)$$

$$P_1 - P_2 = (\rho_{Hg} - \rho_m)\cdot g h$$

Examples

Example$_4$: The flow rate of water at 20°C (ρ= 998 kg/m³ and μ= 1.002*10^{-4} kg/m · s) through a 50 cm diameter pipe is measured with an orifice meter with a 30-cm-diameter opening to be 250 L/s. Determine the pressure difference indicated by the orifice meter and the head loss.

Solution The flow rate of water is measured with an orifice meter. The pressure difference indicated by the orifice meter and the head loss are to be determined.

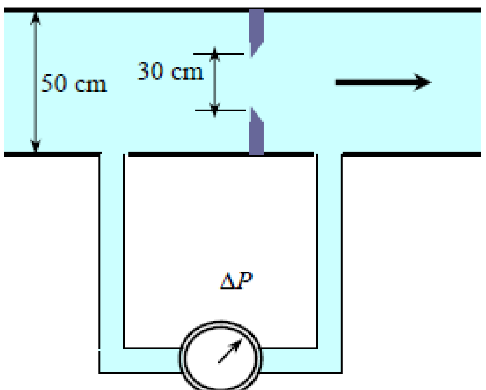
Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the orifice meter is $C_d = 0.61$.

Properties The density and dynamic viscosity of water are given to be ρ = 998 kg/m³ and μ = 1.002×10^{-4} kg/m·s, respectively.

Analysis The diameter ratio and the throat area of the orifice are

$$\beta = d/D = 30/50 = 0.60$$

$$A_0 = \pi d^2/4 = \pi(0.30\text{ m})^2/4 = 0.07069\text{ m}^2$$



For a pressure drop of $\Delta P = P_1 - P_2$ across the orifice plate, the flow rate is expressed as

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

Substituting,

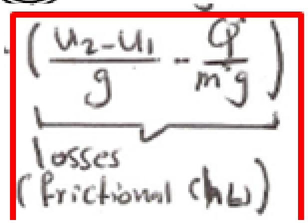
$$0.25\text{ m}^3/s = (0.07069\text{ m}^2)(0.61)\sqrt{\frac{2\Delta P}{(998\text{ kg/m}^3)((1-0.60^4)}}$$

which gives the pressure drop across the orifice plate to be

$$\Delta P = 14{,}600\text{ kg}\cdot\text{m/s}^2 = \mathbf{14.6\ kPa}$$

It corresponds to a water column height of

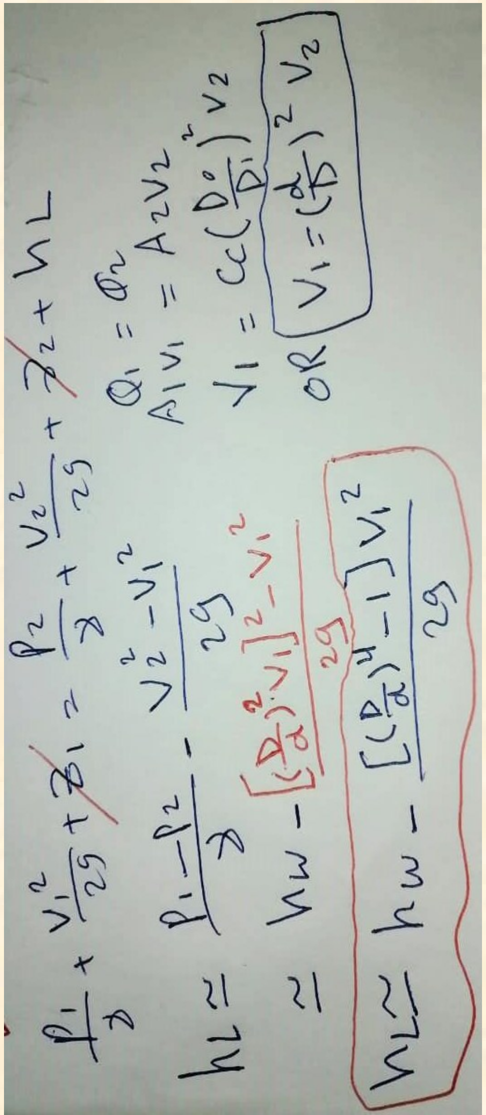
$$h_w = \frac{\Delta P}{\rho_w g} = \frac{14{,}600\text{ kg}\cdot\text{m/s}^2}{(998\text{ kg/m}^3)(9.81\text{ m/s}^2)} = 1.49\text{ m}$$



The head loss between the two measurement sections can also be estimated from the energy equation. Since $z_1 = z_2$, the head form of the energy equation simplifies to

$$h_L \approx \frac{P_1 - P_2}{\rho_f g} - \frac{V_2^2 - V_1^2}{2g} = h_w - \frac{\left[(D/d)^4 - 1\right]V_1^2}{2g} = 1.49\text{ m} - \frac{\left[(50/30)^4 - 1\right](1.27\text{ m/s})^2}{2\left(9.81\text{ m/s}^2\right)} = \mathbf{0.940\ m\ H_2O}$$

where $V_1 = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.250\text{ m}^3/\text{s}}{\pi(0.50\text{ m})^2/4} = 1.27\text{ m/s}$



Examples

Example$_5$: Water is flowing through a Venturi meter whose diameter is 7 cm at the entrance part and 4 cm at the throat. The pressure is measured to be 430 kPa at the entrance and 120 kPa at the throat. Neglecting frictional effects, determine the flow rate of water. *Answer:* 0.0331 m^3/s

Properties We take the density of water to be $\rho = 1000\ \text{kg/m}^3$.

Analysis We take point 1 at the main flow section and point 2 at the throat along the centerline of the venturi meter. Noting that $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

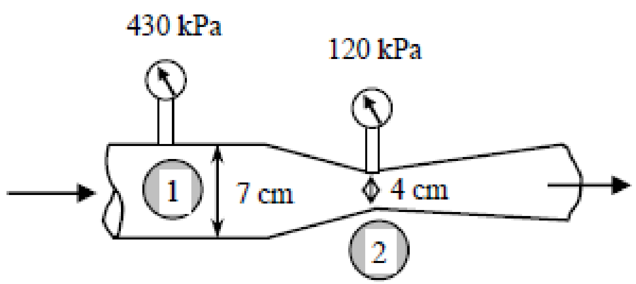
$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad P_1-P_2=\rho\frac{V_2^2-V_1^2}{2} \qquad (1)$$

The flow is assumed to be incompressible and thus the density is constant. Then the conservation of mass relation for this single stream steady flow device can be expressed as

$$\dot V_1=\dot V_2=\dot V \quad\rightarrow\quad A_1V_1=A_2V_2=\dot V \quad\rightarrow\quad V_1=\frac{\dot V}{A_1} \quad\text{and}\quad V_2=\frac{\dot V}{A_2} \qquad (2)$$

Substituting into Eq. (1),

$$P_1-P_2=\rho\frac{(\dot V/A_2)^2-(\dot V/A_1)^2}{2}=\frac{\rho\dot V^2}{2A_2^2}\left(1-\frac{A_2^2}{A_1^2}\right)$$



Solving for $\dot V$ gives the desired relation for the flow rate,

$$\dot V = A_2\sqrt{\frac{2(P_1-P_2)}{\rho[1-(A_2/A_1)^2]}} \qquad (3)$$

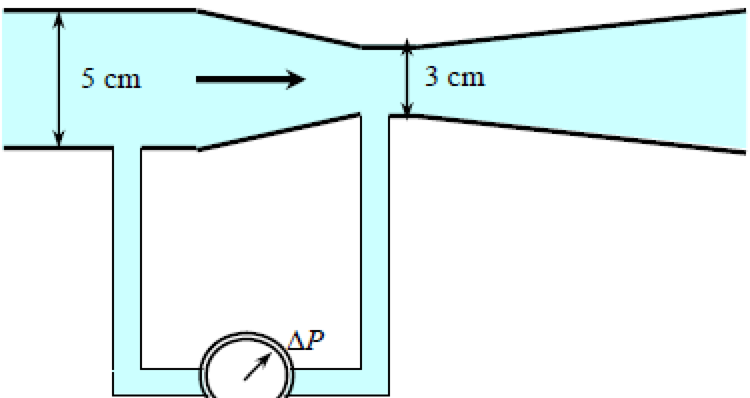
The flow rate for the given case can be determined by substituting the given values into this relation to be

$$\dot V=\frac{\pi D_2^2}{4}\sqrt{\frac{2(P_1-P_2)}{\rho[1-(D_2/D_1)^4]}}=\frac{\pi(0.04\ \text{m})^2}{4}\sqrt{\frac{2(430-120)\ \text{kN/m}^2}{(1000\ \text{kg/m}^3)[1-(4/7)^4]}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\text{kN}}\right)}=\mathbf{0.0331\ m^3/s}$$

Examples

Example$_6$: A Venturi meter equipped with a differential pressure gage is used to measure the flow rate of water at 15°C (ρ= 999.1 kg/m^3) through a 5-cm-diameter horizontal pipe. The diameter of the Venturi neck is 3 cm, and the measured pressure drop is 5 kPa. Taking the discharge coefficient to be 0.98, determine the volume flow rate of water and the average velocity through the pipe. *Answers:* 2.35 L/s and 1.20 m/s

Solution A Venturi meter equipped with a differential pressure gage is used to measure to flow rate of water through a horizontal pipe. For a given pressure drop, the volume flow rate of water and the average velocity through the pipe are to be determined.



Assumptions The flow is steady and incompressible.

Properties The density of water is given to be ρ = 999.1 kg/m^3. The discharge coefficient of Venturi meter is given to be C_d = 0.98.

Analysis The diameter ratio and the throat area of the meter are

$$\beta = d / D = 3/5 = 0.60$$

$$A_0 = \pi d^2 / 4 = \pi (0.03\ \text{m})^2 / 4 = 7.069 \times 10^{-4}\ \text{m}^2$$

Noting that ΔP = 5 kPa = 5000 N/m^2, the flow rate becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$= (7.069 \times 10^{-4}\ \text{m}^2)(0.98)\sqrt{\frac{2 \times 5000\ \text{N/m}^2}{(999.1\ \text{kg/m}^3)((1-0.60^4)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)}$$

$$= \mathbf{0.00235\ m^3/s}$$

which is equivalent to 2.35 L/s. The average flow velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.00235\ \text{m}^3/\text{s}}{\pi (0.05\ \text{m})^2/4} = \mathbf{1.20\ m/s}$$

Examples

Example$_7$: A Venturi meter is a device that is used for measuring the speed of a fluid within a pipe. The drawing shows a gas flowing at speed u_2 through a horizontal section of pipe whose cross-sectional area is A_2 = 0.0700 m^2. The gas has a density of ρ = 1.30 kg/m^3. The Venturi meter has a cross-sectional area of A_1 = 0.0500 m^2 and has been substituted for a section of the larger pipe. The pressure difference between the two sections is P_2 - P_1 = 120 Pa. Find **(a)** the speed u_2 of the gas in the larger, original pipe and **(b)** the volume flow rate Q of the gas.

Sol.$_7$:

Bernoulli equation gives us

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Make the assumption that

$h_1 \approx h_2$

Bernoulli reduces to

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Solve for difference in pressure

$$P_2 - P_1 = \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

Equation of Continuity gives

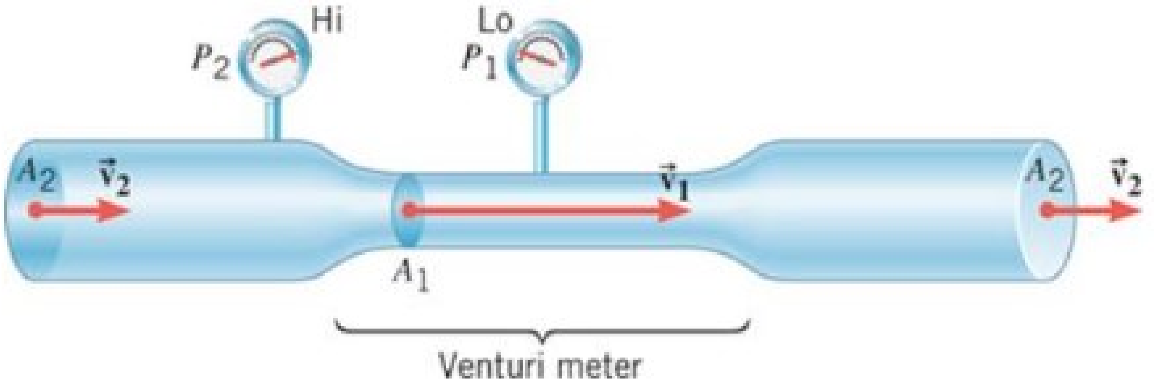
$\rho_1 v_1 A_1 = \rho_2 v_2 A_2$

Assume an incompressible gas and $\rho_1 \approx \rho_2$

$v_1 A_1 = v_2 A_2$

And solving for v_1

$$v_1 = v_2 \frac{A_2}{A_1}$$



http://physics.nmu.edu/~ddonovan/classes/ph201/Homework/Chap11/CH11P69.html

Examples

Sol.$_7$:

$$P_2 - P_1 = \frac{1}{2}\rho\left(\left(v_2\frac{A_2}{A_1}\right)^2 - v_2^2\right) = \frac{1}{2}\rho v_2^2\left(\left(\frac{A_2}{A_1}\right)^2 - 1\right)$$

Now Solve for v_2

$$v_2^2 = \frac{2(P_2 - P_1)}{\rho\left(\left(\frac{A_2}{A_1}\right)^2 - 1\right)}$$

$$v_2 = \sqrt{\frac{2(P_2 - P_1)}{\rho\left(\left(\frac{A_2}{A_1}\right)^2 - 1\right)}} = \sqrt{\frac{2(120\,Pa)}{\left(1.30\ {kg}/{m^3}\right)\left(\left(\frac{0.0700\ m^2}{0.0500\ m^2}\right)^2 - 1\right)}}$$

$$v_2 = \sqrt{\frac{240\,Pa}{\left(1.30\ {kg}/{m^3}\right)(1.96 - 1)}} = \sqrt{192.31\ {m^2}/{s^2}} = 13.87\ {m}/{s}$$

$$Q = Av = A_1v_1 = A_2v_2$$

$$Q = A_2v_2 = (0.0700\ m^2)(13.87\ {m}/{s}) = 0.971\ {m^3}/{s}$$

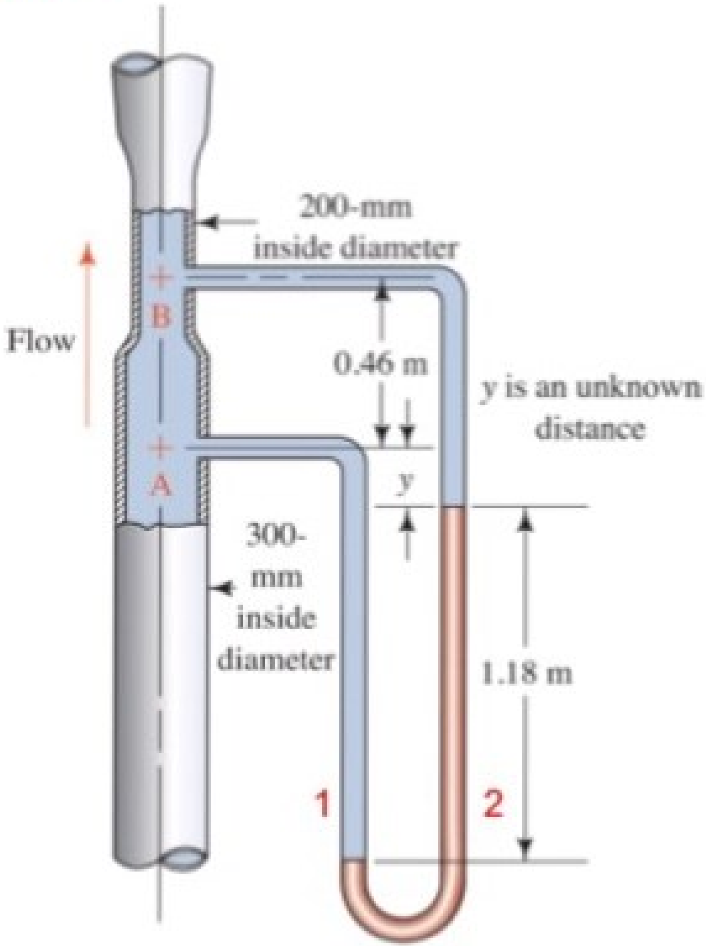
$$v_2 = 14\ {m}/{s}$$
$$Q = 0.97\ {m^3}/{s}$$

Examples

Example$_8$: https://www.chegg.com/homework-help/questions-and-answers/example-problem-venturi-meter-system-venturi-meter-shown-figure-blow-carries-water-60-degr-q16945549

Example Problem: Venturi meter system.

The venturi meter shown in figure blow carries water at 60°C. The inside dimensions are machined to the sizes shown in figure. The specific gravity of the gage fluid in the manometer is 1.25. Calculate the velocity of flow at section A and the volume flow rate of water.



$P_1 = P_A + \gamma_w(y+1.18)$

$P_2 = P_B + \gamma(0.46+y) + 1.25\gamma_w(1.18)$

But, as $P_1 = P_2$

Therefore from the above equations, $P_A - P_B$ can be found. Complete the rest of the solution.

Note: A venturi meter is a device that uses a constriction in a flow system to measure the velocity of flow.

Homeworks (3):

Hw$_1$: The velocity of a fluid flowing in a pipe is to be measured by two different Pitot-type mercury manometers shown in Fig. P5–38C. Would you expect both manometers to predict the same velocity for flowing water? If not, which would be more accurate? Explain. What would your response be if air were flowing in the pipe instead of water?

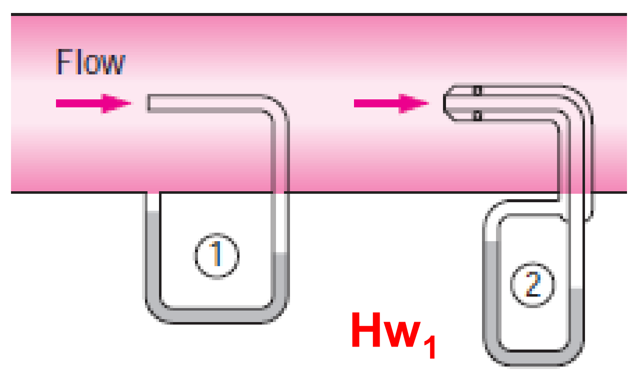


FIGURE P5–38C

Hw$_2$: Explain how flow rate is measured with obstruction type flowmeters. Compare orifice meters, flow nozzles, and Venturi meters with respect to cost, size, head loss, and accuracy.

Homeworks (3):

Hw$_3$: Air is flowing through a venturi meter whose diameter is 2.6 in at the entrance part (location 1) and 1.8 in at the throat (location 2). The gage pressure is measured to be 12.2 sia at the entrance and 11.8 psia at the throat. Neglecting frictional effects, show that the volume flow rate can be expressed as

$$\dot{V} = A_2\sqrt{\frac{2(P_1 - P_2)}{\rho(1 - A_2^2/A_1^2)}}$$

Hw$_4$: The mass flow rate of air at 20°C (ρ =1.204 kg/m^3) through a 15 cm diameter duct is measured with a Venturi meter equipped with a water manometer. The Venturi neck has a diameter of 6 cm, and the manometer has a maximum differential height of 40 cm. Taking the discharge coefficient to be 0.98, determine the maximum mass flow rate of air this Venturi meter can measure. *Answer:* 0.273 kg/s

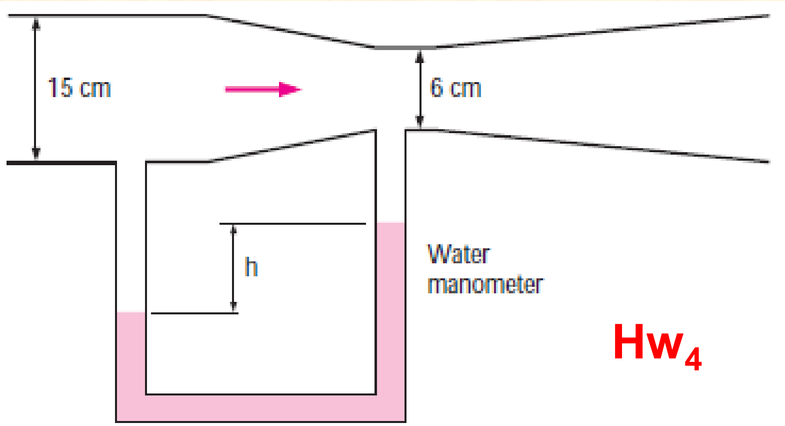


FIGURE P8–105

Homeworks (3):

Hw$_5$: A vertical Venturi meter equipped with a differential pressure gage shown in Fig. P8–107 is used to measure the flow rate of liquid propane at 10°C (ρ = 514.7 kg/m^3) through an 8-cm-diameter vertical pipe. For a discharge coefficient of 0.98, determine the volume flow rate of propane through the pipe.

Hw$_6$: A flow nozzle equipped with a differential pressure gage is used to measure the flow rate of water at 10°C (ρ = 999.7 kg/m^3 and μ= 1.307*10^{-3} kg/m · s) through a 3cm diameter horizontal pipe. The nozzle exit diameter is 1.5 cm, and the measured pressure drop is 3 kPa. Determine the volume flow rate of water, the average velocity through the pipe, and the head loss.

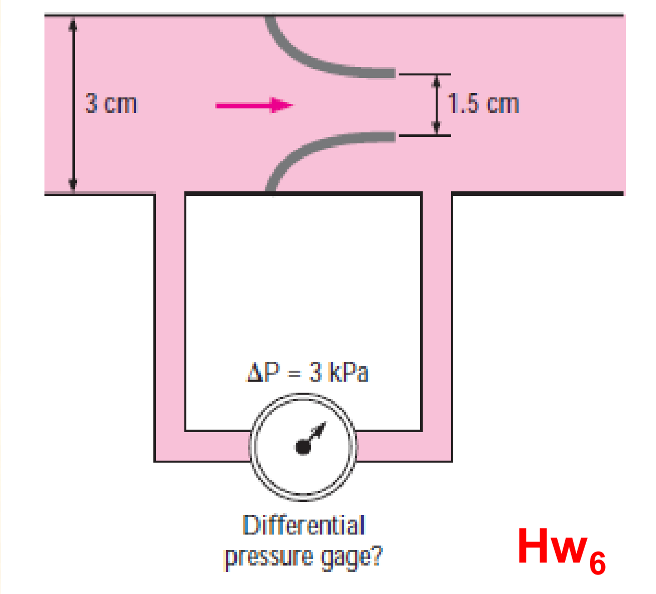


FIGURE P8–108

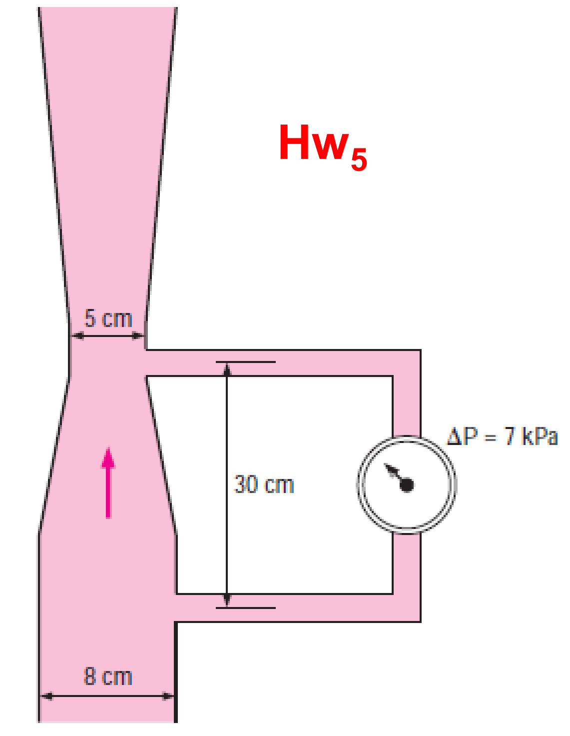


FIGURE P8–107

- Note: Solve all six Homeworks and sending me the answering next week on Thursday 27 February 2025.

- I hope everything is clear for all students

Good luck